EE 232: Lightwave Devices
Lecture #18 – LED modulation rate - Small signal analysis

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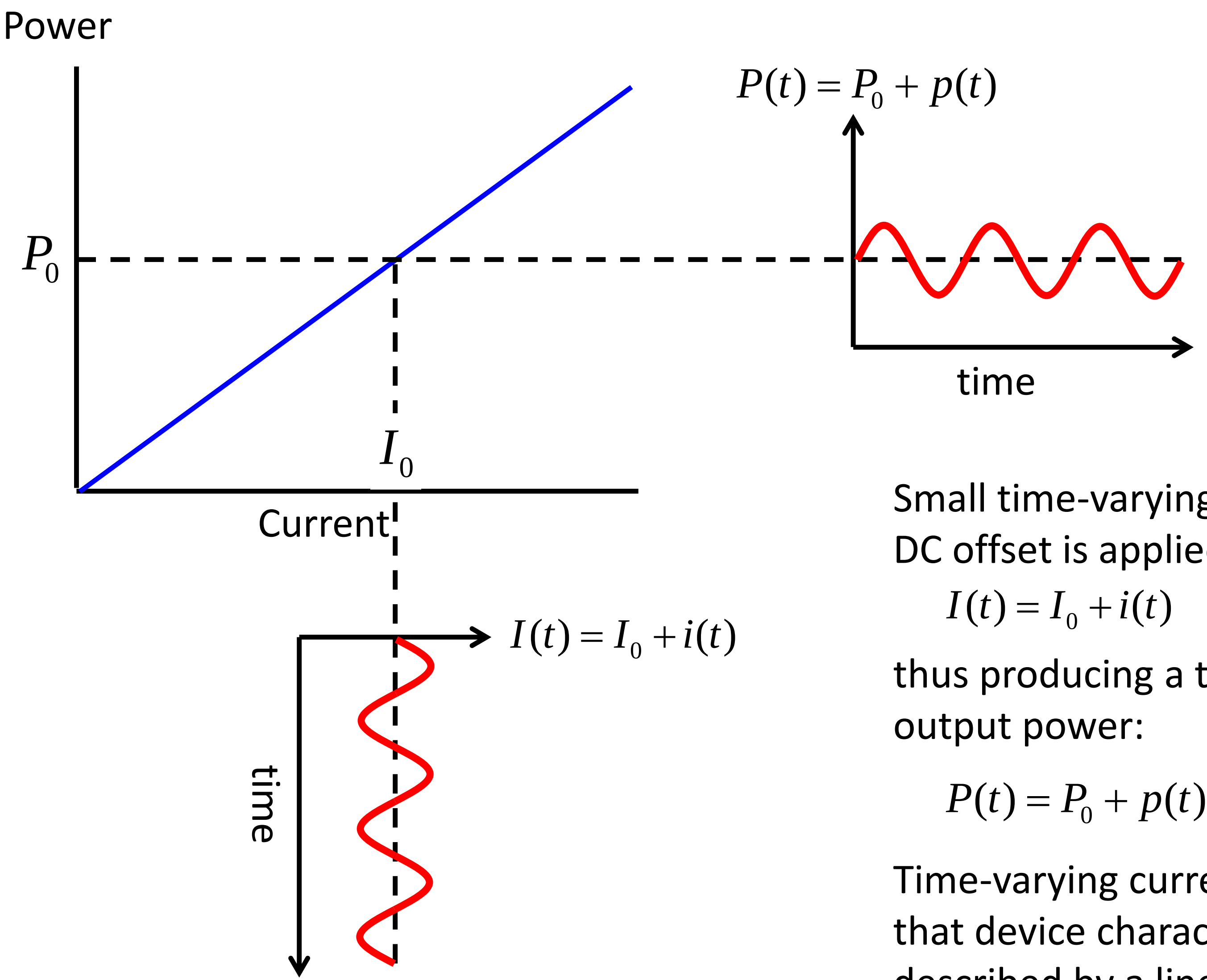
Small signal analysis



Small time-varying current with DC offset is applied to the device:

$$I(t) = I_0 + i(t)$$

thus producing a time dependent output power:

$$P(t) = P_0 + p(t)$$

Time-varying current is small such that device characteristic can be described by a linear extrapolation away from the bias point.

Small signal analysis - LED

Rate equation analysis

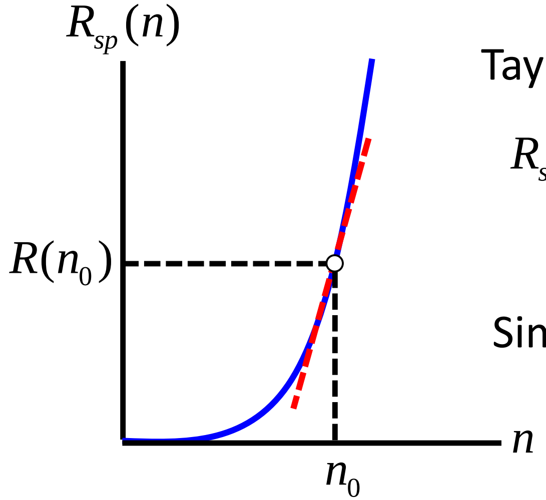
$$\frac{dn(t)}{dt} = \eta_i \frac{J(t)}{qd} - R_{SRH}(t) - R_{sp}(t) - R_{Auger}(t)$$

$$\frac{dn(t)}{dt} \simeq \eta_i \frac{J(t)}{qd} - An(t) - Bn^2(t) - Cn^3(t)$$

Let $n(t) = n_0 + \Delta n(t)$

$$J(t) = J_0 + \Delta J(t)$$

$$P(t) = P_0 + \Delta P(t)$$



Taylor expansion

$$R_{sp}(t) \simeq R(n_0) + \left.\frac{\partial R(n)}{\partial n}\right|_{n=n_0} \Delta n(t)$$

$$= Bn_0^2 + 2Bn_0\Delta n(t) = Bn_0^2 + \Delta n(t)/\tau_{sp}$$

Similarly,

$$R_{SRH}(t) \simeq An_0 + A\Delta n(t) = An_0 + \Delta n(t)/\tau_{SRH}$$

$$R_{Auger}(t) \simeq Cn_0^3 + 3Cn_0^2\Delta n(t) = Cn_0^3 + \Delta n(t)/\tau_{Auger}$$

Small signal analysis - LED

$$\frac{d[n_0 + \Delta n(t)]}{dt} = \eta_i \frac{J_0 + \Delta J(t)}{qd} - An_0 - Bn_0^2 - Cn_0^3 - \frac{\Delta n(t)}{\tau}$$

$$\boxed{\frac{d\Delta n(t)}{dt} = \eta_i \frac{\Delta J(t)}{qd} - \frac{\Delta n(t)}{\tau}}$$

where $\tau^{-1} = \tau_{SRH}^{-1} + \tau_{sp}^{-1} + \tau_{Auger}^{-1} = A - 2Bn_0 - 3Cn_0^2$

Similarly, $P(t) = h\nu R_{sp}(t) V_{act}$

$$P_0 + \Delta P(t) = h\nu V_{act}\left[Bn_0^2 + \frac{\Delta n(t)}{\tau_{sp}}\right]$$

$$\boxed{\Delta P(t) = \frac{\Delta n(t)}{\tau_{sp}} h\nu V_{act}}$$

Sinusoidal excitation - LED

We assume the excitation is sinusoidal

$$\Delta n(t) = \mathrm{Re}[\Delta n(\omega) e^{-i\omega t}]$$

$$\Delta J(t) = \mathrm{Re}[\Delta J(\omega) e^{-i\omega t}]$$

$$\Delta P(t) = \mathrm{Re}[\Delta P(\omega) e^{-i\omega t}]$$

Then,

$$\frac{d}{dt}\Delta n e^{-i\omega t} = \eta_i \frac{\Delta J e^{-i\omega t}}{qd} - \frac{\Delta n e^{-i\omega t}}{\tau}$$

$$-\Delta n(i\omega) = \eta_i \frac{\Delta J}{qd} - \frac{\Delta n}{\tau}$$

$$\Delta n\left[\tau^{-1} - i\omega\right] = \eta_i \frac{\Delta J}{qd}$$

$$\Delta n = \eta_i \frac{\Delta J}{qd} \frac{1}{\tau^{-1} - i\omega}$$

$$\boxed{\Delta n = \eta_i \frac{\Delta J \tau}{qd} \frac{1}{1 - i\omega\tau}}$$

Similarly,

$$\Delta P e^{-i\omega t} = \frac{\Delta n e^{-i\omega t}}{\tau_{sp}} h\nu V_{act}$$

$$\Delta P = h\nu V_{act} \eta_i \frac{\tau}{\tau_{sp}} \frac{\Delta J}{qd} \frac{1}{1 - i\omega\tau}$$

$$\Delta P = \eta_i \frac{\tau}{\tau_{sp}} \frac{1}{1 - i\omega\tau} \frac{h\nu}{q} \Delta I$$

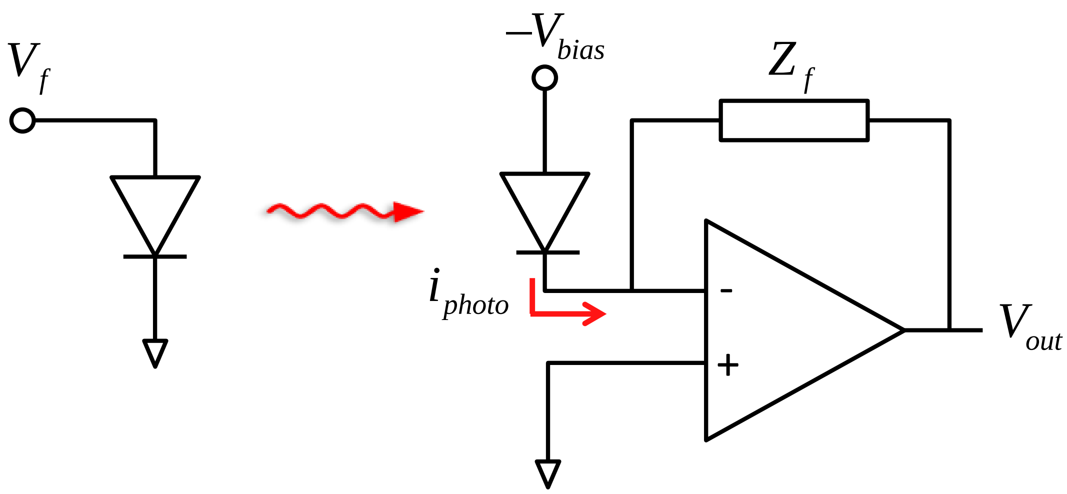
$$\boxed{\frac{\Delta P}{\Delta I} = \eta_i \frac{\tau}{\tau_{sp}} \frac{h\nu}{q} \left(\frac{1}{1 - i\omega\tau}\right)}$$

3dB-frequency

Optical 3dB-point: Frequency in which the **optical signal power** is reduced by one-half from the DC power.

$$\left|\frac{\Delta P}{\Delta I}\right|_{\omega=2\pi f_{3dB},opt} = 0.5\left|\frac{\Delta P}{\Delta I}\right|_{\omega=0} \rightarrow \frac{1}{\sqrt{1+(2\pi f_{3dB,opt})^2\tau^2}} = 0.5 \qquad f_{3dB,opt} = \frac{\sqrt{3}}{2\pi}\frac{1}{\tau}$$

Electrical 3dB-point: Frequency in which the **electrical signal power** is reduced by one-half from the DC power.



$$P_{elec} \propto (i_{photo}^2) \propto (P^2)$$

$$\therefore \frac{1}{1+(2\pi f_{3dB})^2\tau^2} = 0.5$$

$$\boxed{f_{3dB} = \frac{1}{2\pi}\frac{1}{\tau}}$$

3dB-frequency

Let's write 3dB-frequency in terms of the injected current.

$$f_{3dB} = \frac{1}{2\pi}\frac{1}{\tau}$$

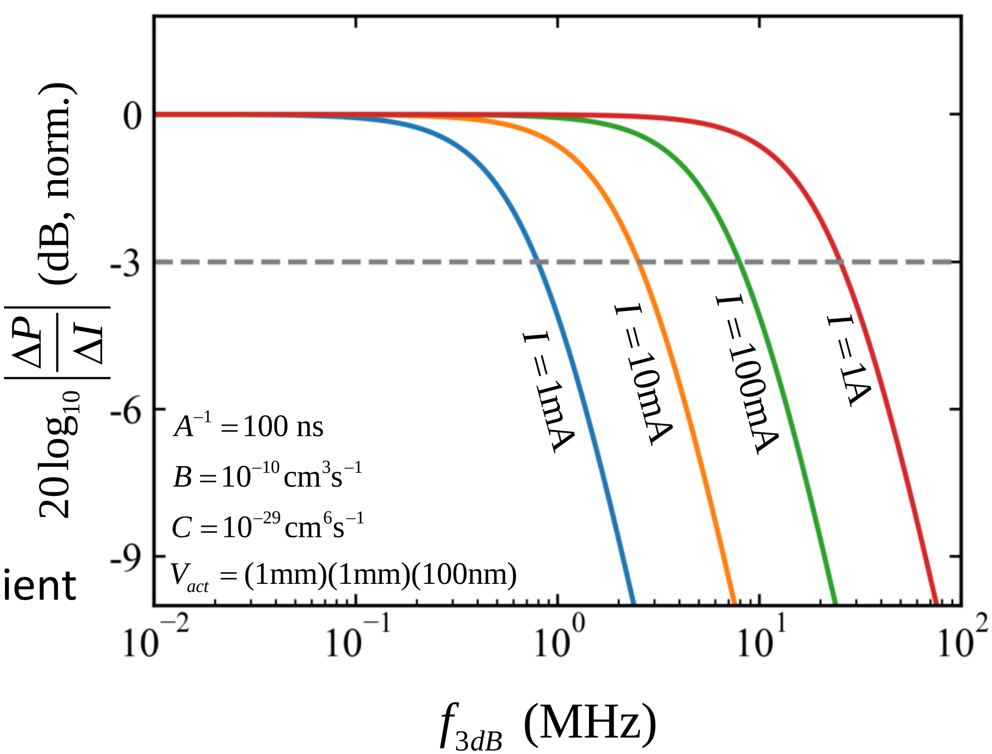
$$= \frac{1}{2\pi}\left(A + 2Bn + 3Cn^2\right) \simeq \frac{1}{\pi}Bn$$

(Assuming that spontaneous emission dominates)

Since, $\dfrac{\eta_i J}{qd} \simeq Bn^2$

$$f_{3dB} \simeq \frac{1}{\pi}\sqrt{\frac{B\eta_i}{qd}}\sqrt{J}$$

$$\boxed{f_{3dB} = \frac{1}{\pi}\sqrt{\frac{B\eta_i}{qV_{act}}}\sqrt{I}}$$



For highest speed:

(1) Large spontaneous emission coefficient
(2) Maximize drive current
(3) Small active region volume